Master MAGIS

Adhesive contacts :
A fracture mechanics approach

Antoine Chateauminois
ESPCI/SIMM
CNRS UMR 7615
10 rue Vauquelin – 75231 Paris Cedex 05

antoine.chateauminois@espci.fr

Introduction

Contact mechanics: which problems ?

- Tribological applications

  Friction control, lubrication

  Damage : cracking, wear

  Assesment of surface treatments, coatings (Hardness measurement)

- Characterization of surfaces and interfaces

  Measurement of adhesive properties

  Physical interaction between surfaces(Van der Waals, capillarity, physal chemistry…)

  Méchanics of surface (plasticity, viscoelasticity…)

  Rheology of confined layers

Introduction

Several intricate fields:

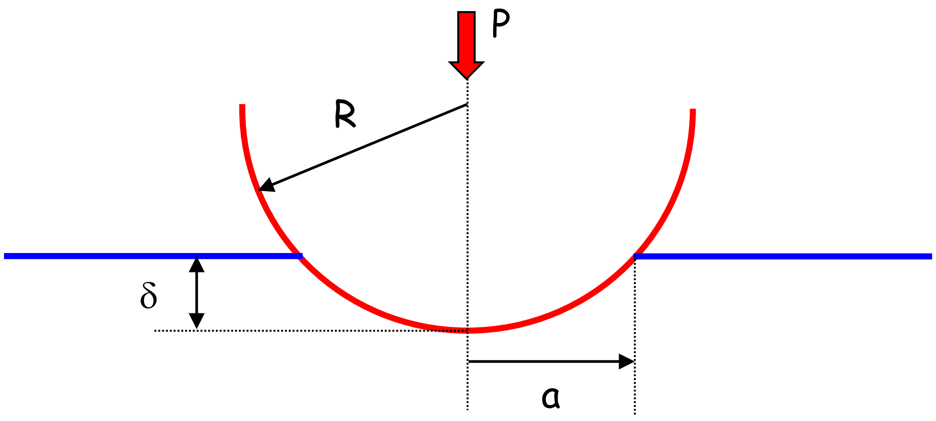
- Mechanics of continuous medium
  Mixed boundary conditions, fracture
- Mechanics of materials
  Elasticity, viscoelasticity, plasticity, toughness
- Surface interactions
  Van der Waals, capillary forces, Chemical Physics….
- Fluid mechanics
  Hydrodynamic , elasto-hydrodynamic lubrication

Heinrich Rudolf Hertz
(1857-1894)

A wide range of problems and scales:

- Materials mechanical properties
  E ~ 10kPa E ~ 200 GPa
- Surface properties
  Physical-chemistry, environnement
- Loading conditions and geometry of the contacting bodies

Hetz theory - scaling approach /I



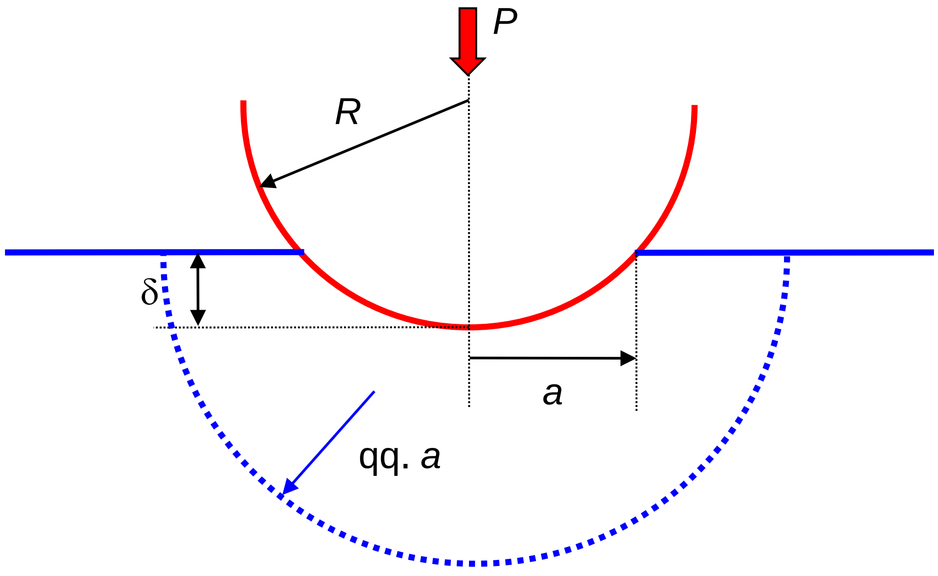
- **Isotrope semi-infinite elastic body**
  Young's modulus *E*

- **Rigid spherical indenter**
  Radius of curvature *R*

Equilibrium relationship under applied load, P, indentation depth, δ and contact radius, a

Energy balance

Hetz theory – scaling approach /II



- **Average deformation within the contact:**

$$\varepsilon \sim \delta / a$$

- **Geometry**

$$\delta \sim a^2/R \qquad (\delta << a)$$

$\Rightarrow$ $\varepsilon \sim a/R$

Total energy of the system:

$$U \sim -P\delta + E(\delta/a)^2 a^3$$

Ext. work ($-P\delta$), ε (δ/a), volume (a^3)

$dU/d\delta = 0 \Rightarrow$

$$P \sim a^3 E / R$$
$$\delta \sim a^2 / R$$

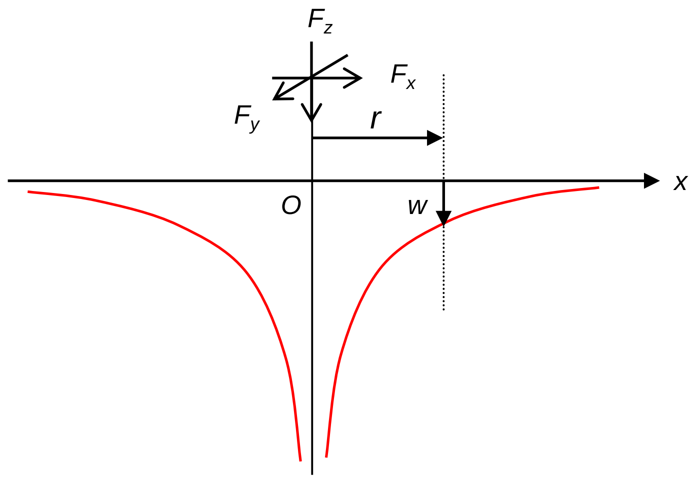
Hertz contact theory (1882) /I

Hypothesis :

- Homogeneous, isotrope, semi-infinite bodies (modulus E, Poisson's ration ν)
- Non adhesive purely elastic contact
- Frictionless (no shear stress at the interface)
- Surfaces are described locally by their radii of curvature
- $a \ll R$ (small strain, non conformal contacts)

Hertz contact theory / II

POINT LOADING OF A SEMI-INFINITE MEDIUM : The Green's Tensor

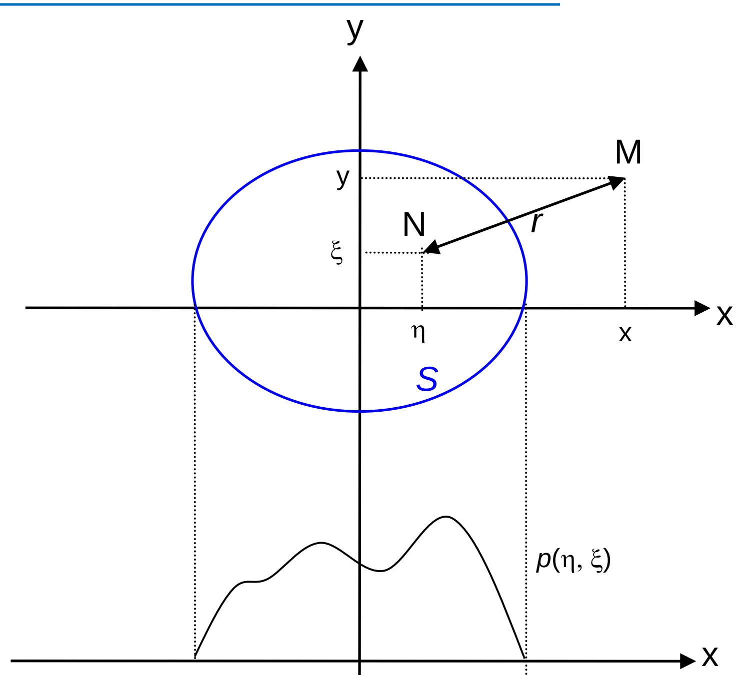


$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

Vertical displacement *w(r)* of a surface point :

$$w(r) = \frac{1-\nu^2}{\pi E}\frac{P}{r}$$

CONTACT PRESSURE DISTRIBUTION



Point loading at *N:*

$$f_z = p(\eta,\zeta)d\eta d\zeta$$

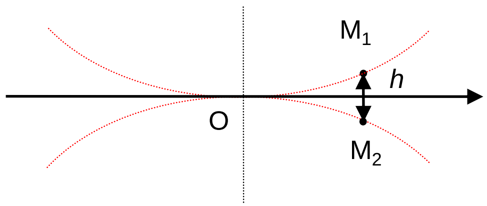
Total vertical displacement at *M* induced by the pressure distribution:

$$w(x,y) = \frac{1-\nu^2}{\pi E}\iint_S \frac{p(\eta,\xi)d\eta d\xi}{\sqrt{(x-\eta)^2+(y-\xi)^2}}$$

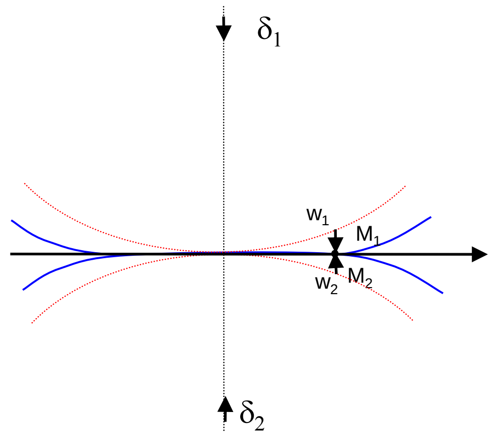
$$r = \sqrt{(x-\eta)^2+(y-\xi)^2}$$

Hertz contact theory / III

Contact equations



P = 0



P > 0

$$h = \frac{x^2}{2R_1} + \frac{y^2}{2R_2}$$

$$R_1 = \frac{R_1^{'} R_1^{''}}{R_1^{'} + R_1^{''}} \qquad R_2 = \frac{R_2^{'} R_2^{''}}{R_2^{'} + R_2^{''}}$$

Within the contact:

$$w_1(x,y) + w_2(x,y) = \delta - \frac{x^2}{2R_1} - \frac{y^2}{2R_2}$$

$$\frac{1}{\pi}\left(\frac{1-\nu_1^{\,2}}{E_1} + \frac{1-\nu_2^{\,2}}{E_2}\right)\iint_S \frac{p(\eta,\xi)\,d\eta\, d\xi}{\sqrt{(x-\eta)^2 + (y-\xi)^2}} = \delta - \frac{x^2}{2R_1} - \frac{y^2}{2R_2}$$

⇒

$$p(x,y) = p_0\sqrt{\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)}$$

Semi-elliptical pressure distribution

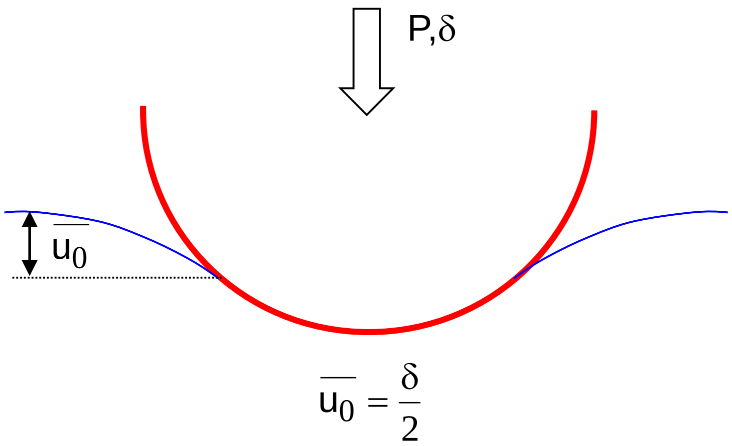
Hertz contact theory / IV

- **Solution for a sphere-on-flat contact**

$$P = \frac{a^3 K}{R}$$

$$\delta = \frac{a^2}{R}$$

$$p_0 = \frac{3}{2}\frac{p}{\pi a^2} = \frac{3}{2} p_m = \left[\frac{6PE^{*2}}{\pi^3 R^2}\right]^{1/3}$$

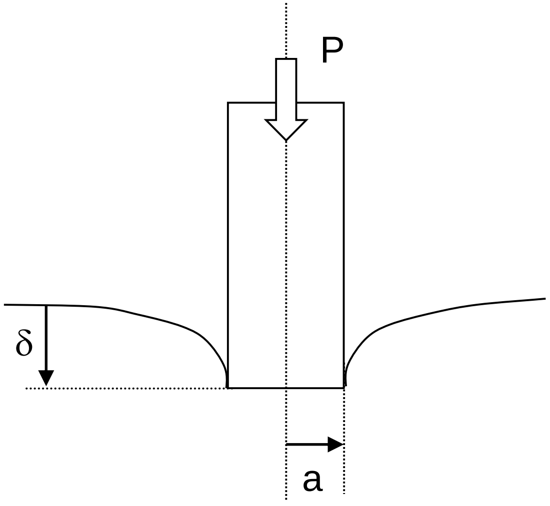


$$K = \frac{4}{3}E^*$$

$$\frac{1}{E^*} = \frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Contact with a flat-ended cylindrical punch



Frictionless

- Approximate approach:

$$P \sim a E \delta$$

- Exact solution (Boussinesq,1885) :

$$P = 3/2\ a K \delta$$

- Normal contact stiffness with an axisymetric indenter:

$$k = dP / d\delta$$

$$k = 3/2\ aK$$

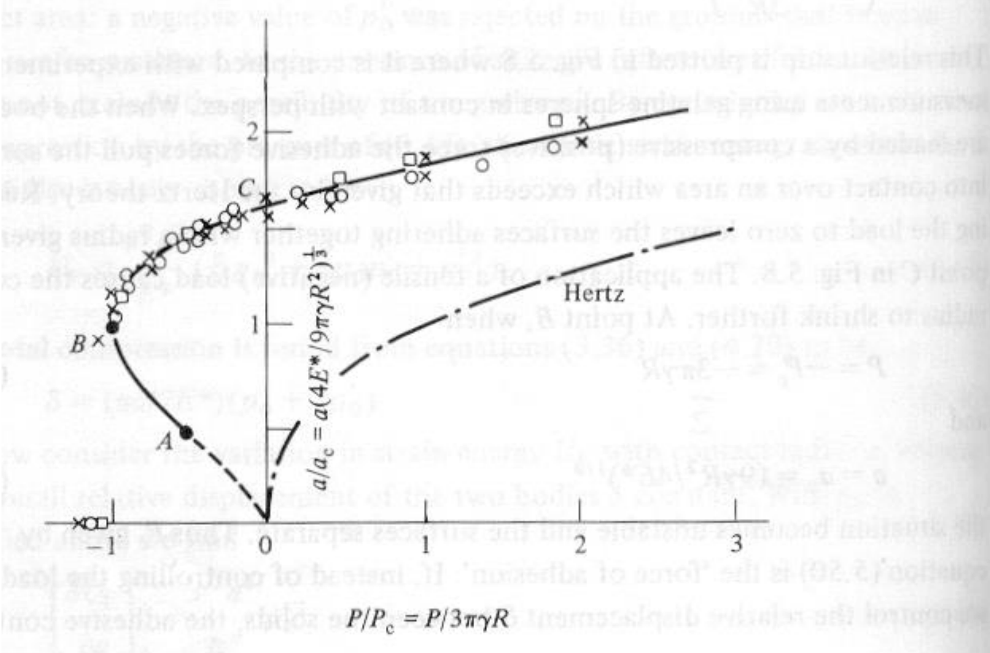
Vertical slope of the free surface at contact edge
Divergence of the stresses at contact edge
→ Relationship with fracture mechanics

$$p(r) = p_0\left(1 - \frac{r^2}{a^2}\right)^{-1/2}$$

ADHESIVE CONTACT

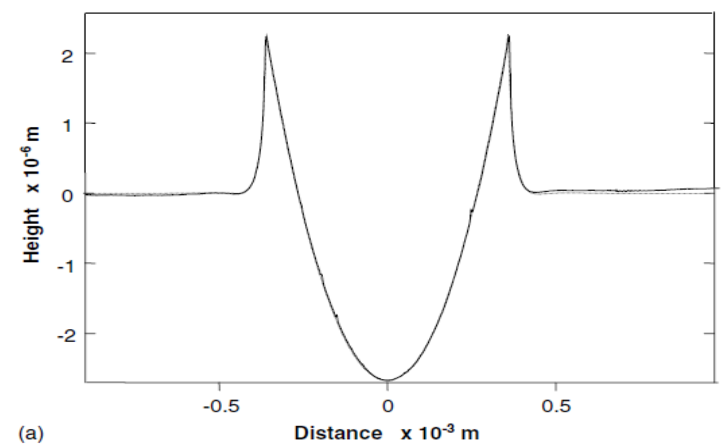
- Contact between gelatin spheres and a flat PMMA surface

Fig. 5.8. Variation of contact radius with load, eq. (5.49), compared with measurements on gelatine spheres in contact with perspex. Radius R: circle – 24.5 mm, cross – 79 mm, square – 255 mm.



Johnson, K. Kendall, K. & Roberts, A.
Surface energy and the contact of elastic solids.
Proceedings of the Royal Society of London A, **1971**, *324*, 301-313

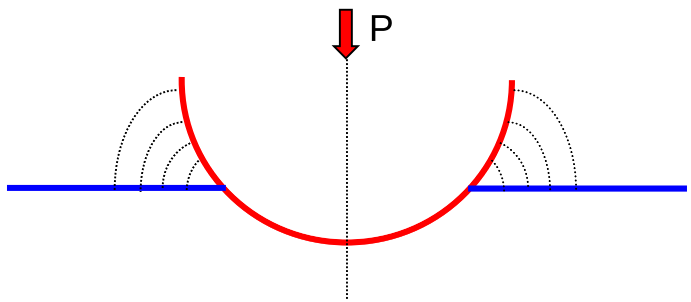
- Surface profile of a contact between a glass lens and an adhesive acrylate coating



Mary, P. Chateauminois, A. &. Frétigny Christian
Contact deformation of elastic coatings in adhesive contacts with spherical probes.
Journal of Applied Physics, **2006**, *39*, 3665-3673

Reversible thermodynamic work of adhesion (Dupré)

Surface forces (VdW) outside the contact:

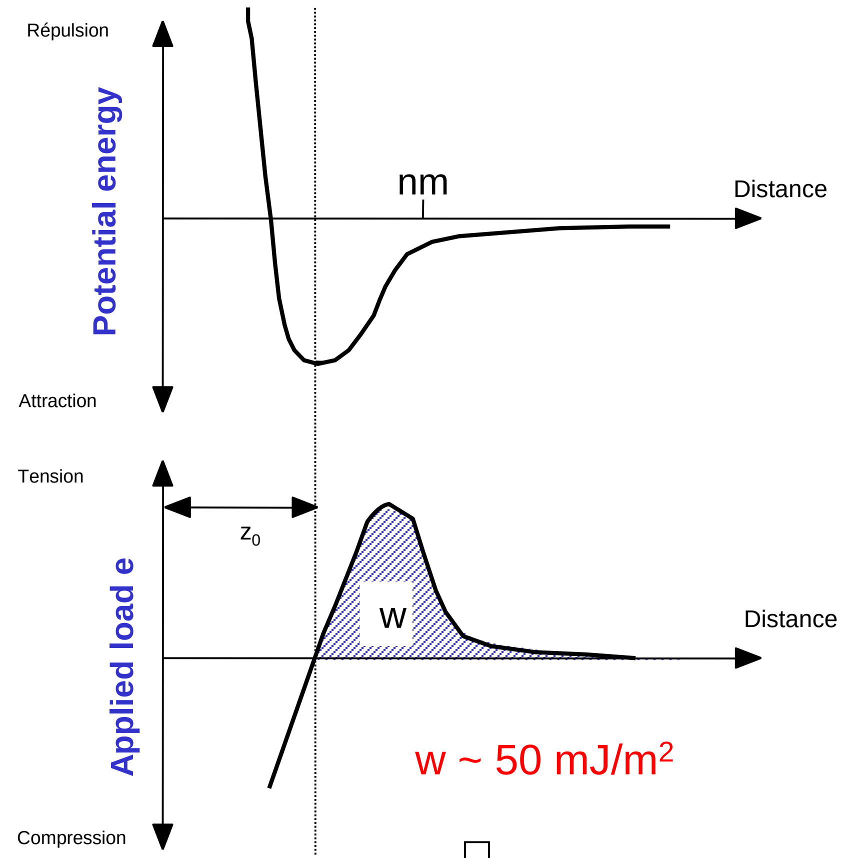


Self-consistency of the problem:

Interaction forces

⇕

Surface deformation



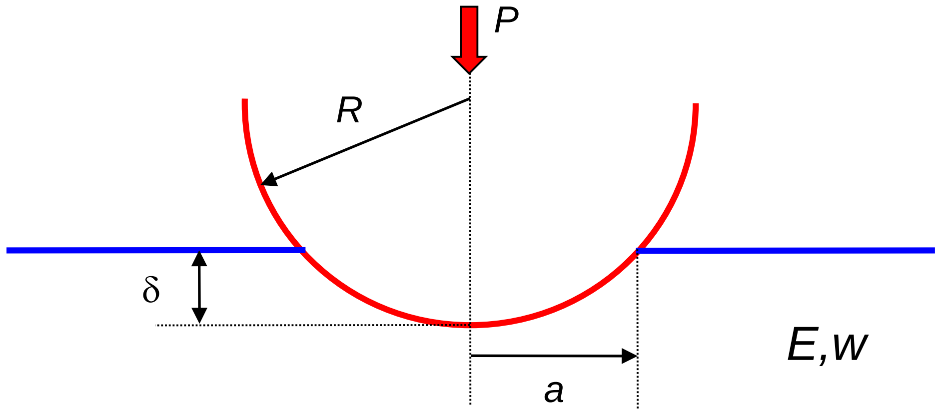
Equilibrium conditions

No chemistry

Reversible thermodynamic work of adhesion (Dupré)

$$W = \gamma_1 + \gamma_2 - \gamma_{12}$$

Approximate theory



- Energy of the system:

$$U \sim -P\,\delta + E\,(\delta/a)^2\,a^3 - w\,a^2$$ with $$\delta = a^2/R$$

- 'Effective' Hertz load:

$$P_{eff} = P + w\,R \sim a^3\,E\,/\,R$$

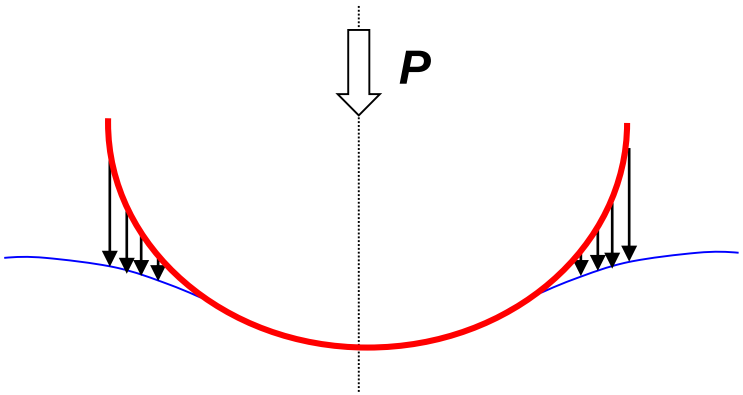
DMT approximation (Derjaguin, Muller, Toporov, 1975)

- Under the action of surface forces, the deformation of the surfaces still obeys Hertz elastic equations.

- Surface forces are assumed to act outside of the contact area only within an external annular zone. Their action is neglected within the contact area.

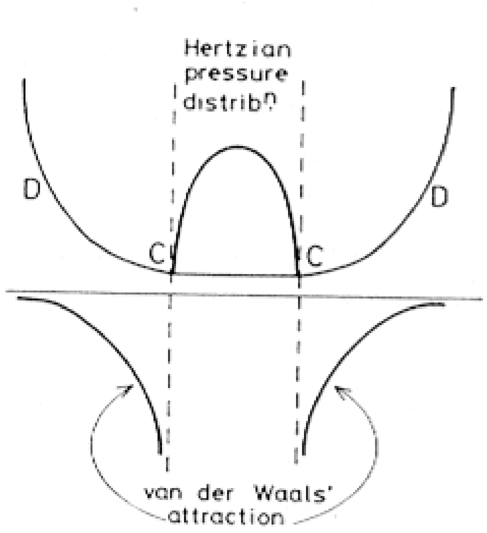
- Surfaces forces do not modify the Hertzian shape of the deformed surfaces.

Rigid materials only



$$P + 2\pi w R = a^3 \; K / R$$

$$\delta = a^2 / R$$



JKR approximation (Johnson, Kendall, Roberts, 1971) / I

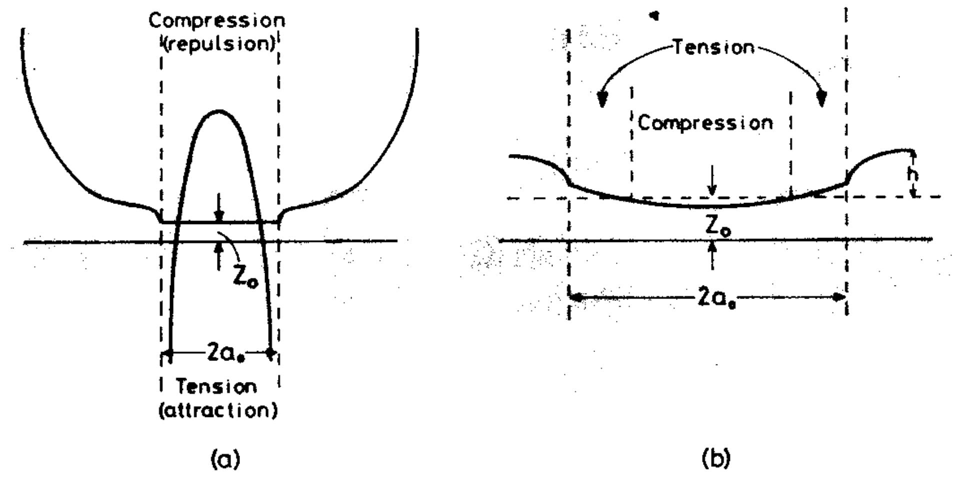
- Developed in order to account of the adhesion of soft materials (E ~ kPa-MPa)

- The range of the surface forces is negligible as compared to the gap between the surfaces outside the contact zone:

$$\sigma_{zz} = 0 \qquad (r > a)$$

- Traction forces are encountered at the periphery of the contact

Soft materials

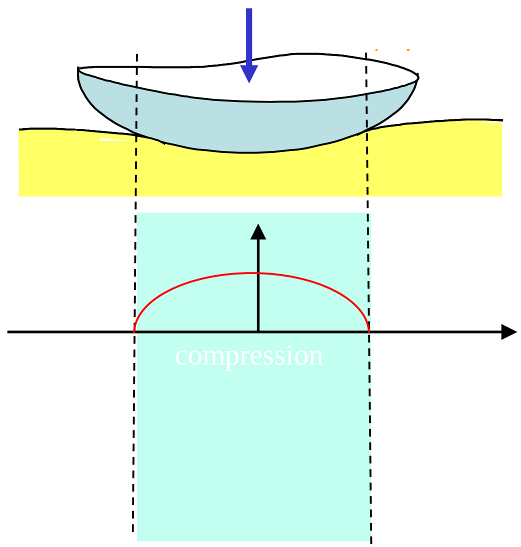


L'approximation JKR (Johnson, Kendall, Roberts, 1971)

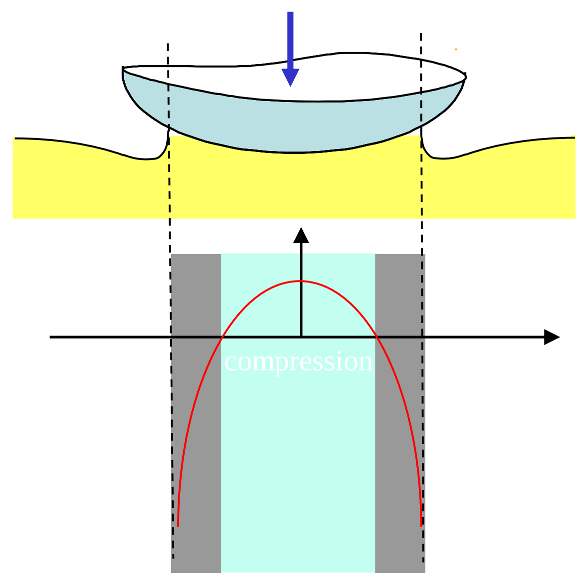
- Adhésion de matériaux mous (*E* ~ kPa-MPa)

- La portée des forces de surface est négligeable devant la séparation des surfaces en bordure de contact:

- Contraintes de traction à la périphérie du contact

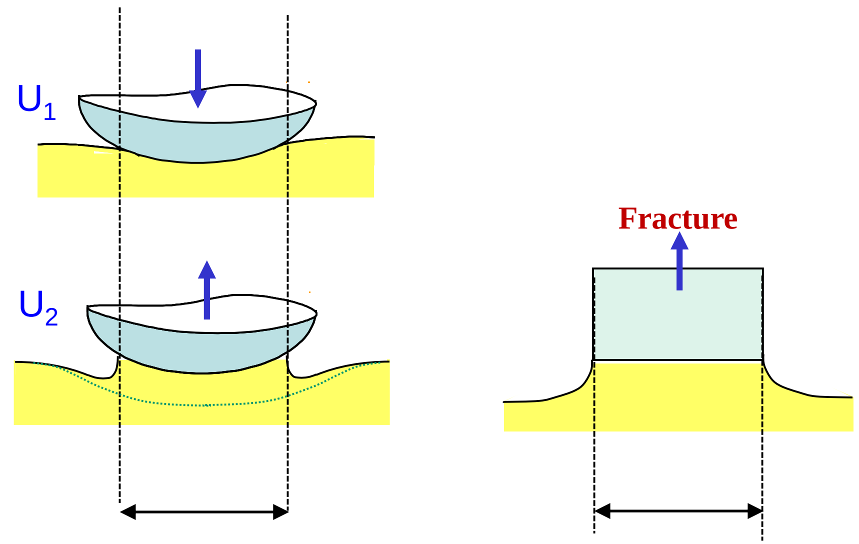


Contact hertzien



Contact JKR

Approximation JKR : détermination de l'état d'équilibre adhésif



Equilibre adhésif atteint en deux étapes:

- (U_1) un contact non adhésif correspondant au rayon d'équilibre atteint sous une charge hertzienne $P_1=a^3K/R$.

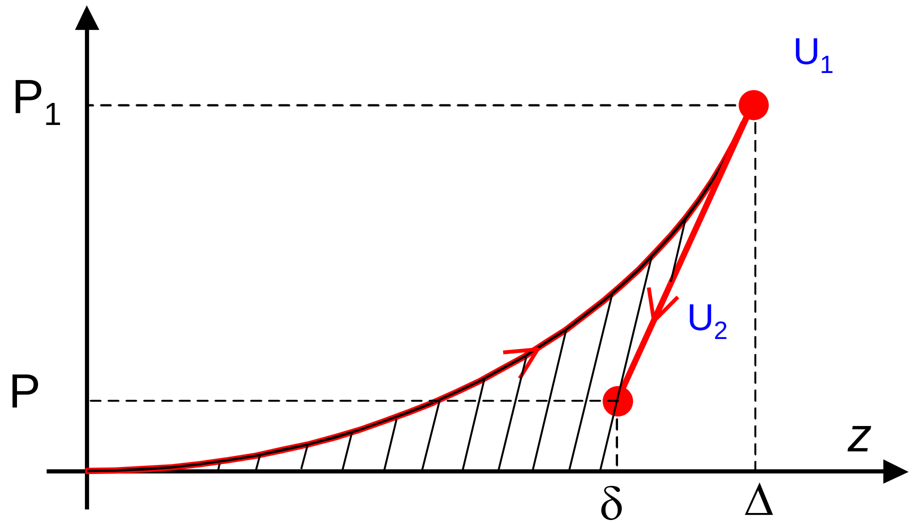
- (U_2) Un déplacement vertical rigide est appliqué à tous les points du contact pour atteindre, à rayon de contact constant *a*, la force *P* et l'indentation δ à l'équilibre.

Condition d'équilibre (Critère de Griffith's)

$$G = \left(\frac{\partial U_E}{\partial A}\right)_\delta = w$$

$$w = \frac{\left(\frac{a^3K}{R} - P\right)^2}{6\pi a^3 K}$$

Equilibre adhésif



JKR Approximation / II

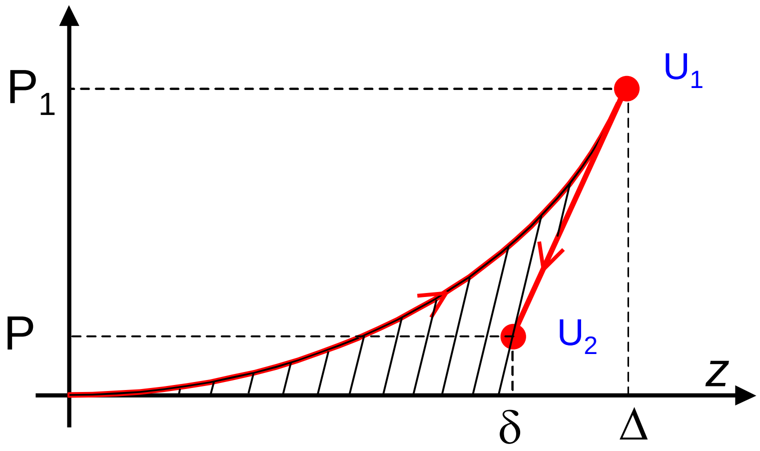
Adhesive equilibrium is achieved into two steps :

- (U_1) a non adhesive contact with the equibrium radius *a* is achieved under the Hertz load $P_1 = a^3K/R$.

- (U_2) A rigid vertical displacement is applied to all the surface points within the contact in order to achieve, at constant contact radius *a*, the equilibrium load *P* and indentation δ

$$U_E = U_1 - U_2 = \underbrace{\int_0^{P_1} \frac{2}{3}\left(\frac{P^2}{K^2 R}\right)^{1/3} dP}_{\text{Hertz}} - \underbrace{\int_{P_1}^{P} \frac{2}{3}\frac{P}{Ka} dP}_{\text{Boussinesq}}$$

$$\left(\frac{\partial U_E}{\partial A}\right)_\delta = \frac{(P_1 - P)^2}{6\pi a^3 K}$$



Equilibrium condition (Griffith's criterion)

$$\left(\frac{\partial U_E}{\partial A}\right)_\delta = w \qquad w = \frac{\left(\frac{a^3K}{R} - P\right)^2}{6\pi a^3 K}$$

JKR Approximation / III

- **Load**

$$P = \underbrace{\frac{a^3 K}{R}}_{\text{Hertzian term } (w=0)} \underbrace{- 3w\pi R \pm \sqrt{6w\pi RP + (3w\pi R)^2}}_{\text{Adhesive term}}$$

- **Indentation :**

$$\delta = \frac{a^2}{R} - \sqrt{\frac{8\pi wa}{3K}}$$

$\delta = a^2/3R + 2P/3aK$

- **Contact radius:**

$$a^3 = \frac{R}{K}\left(P + 3w\pi R + \sqrt{6w\pi RP + (3w\pi R)^2}\right)$$

Comparison JKR / DMT

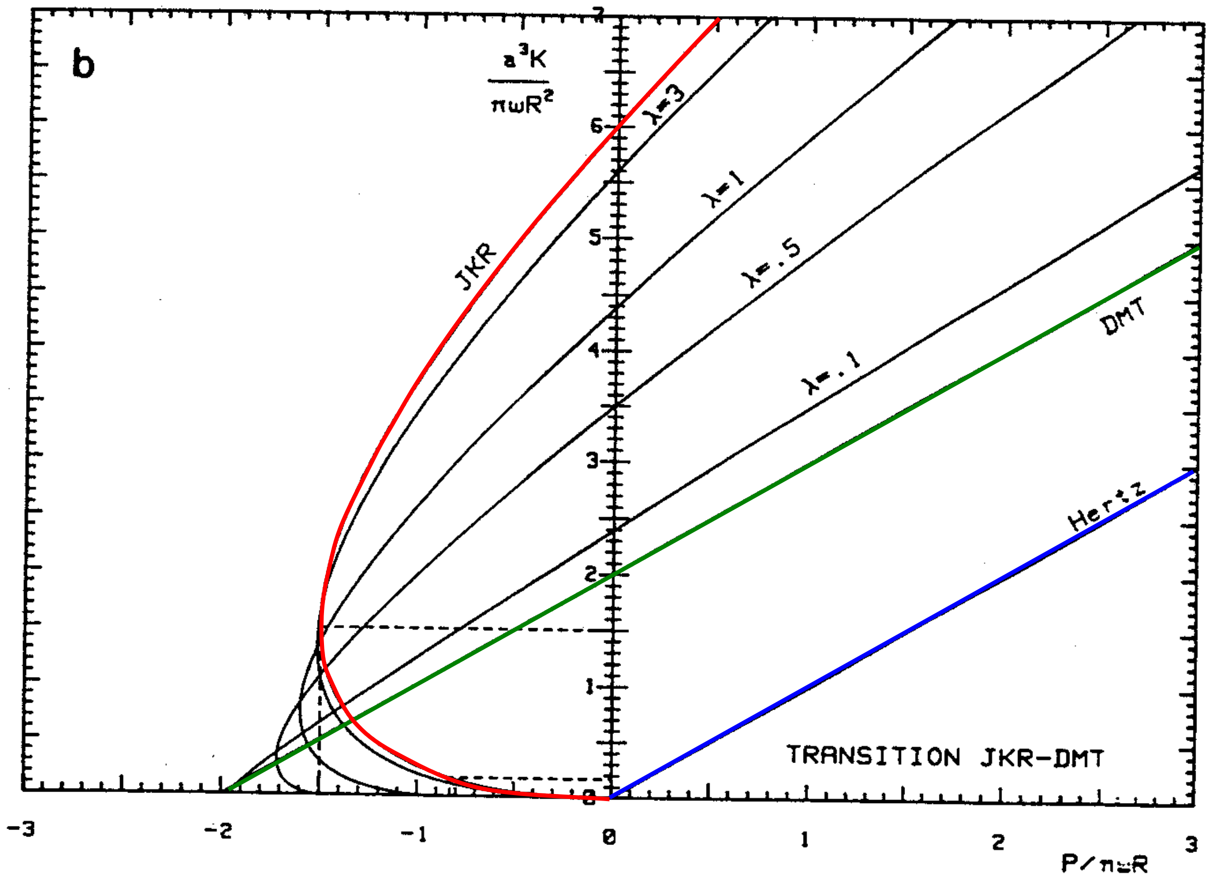
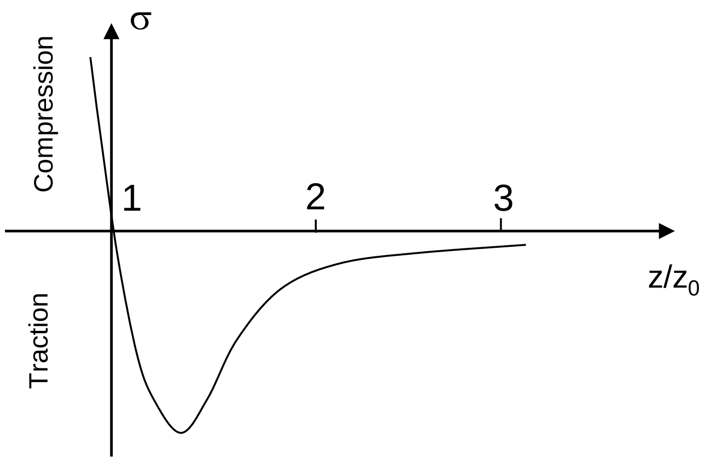


FIG. 5. Radius of contact (a) or cube of the radius of contact (b) plotted against $P/\pi wR$, for various λ. The Hertz curve is shown for comparison.

Validity of the DMT et JKR approximations: The Tabor's parameter



When the gap *h* between the surfaces becomes of the order of magnitude of the range z_0 of surface forces, the action of surface forces outside the contact can no longer be neglected

$$\mu \equiv \left(\frac{Rw^2}{E^{*2} z_0^3} \right)^{1/3} \sim \frac{\text{Typical elastic displacement at the onset of pull-off } h}{\text{Range of surface forces } z_0}$$

- $\mu < 0.1$ High Young's modulus, low adhesion energy, small radii of curvature

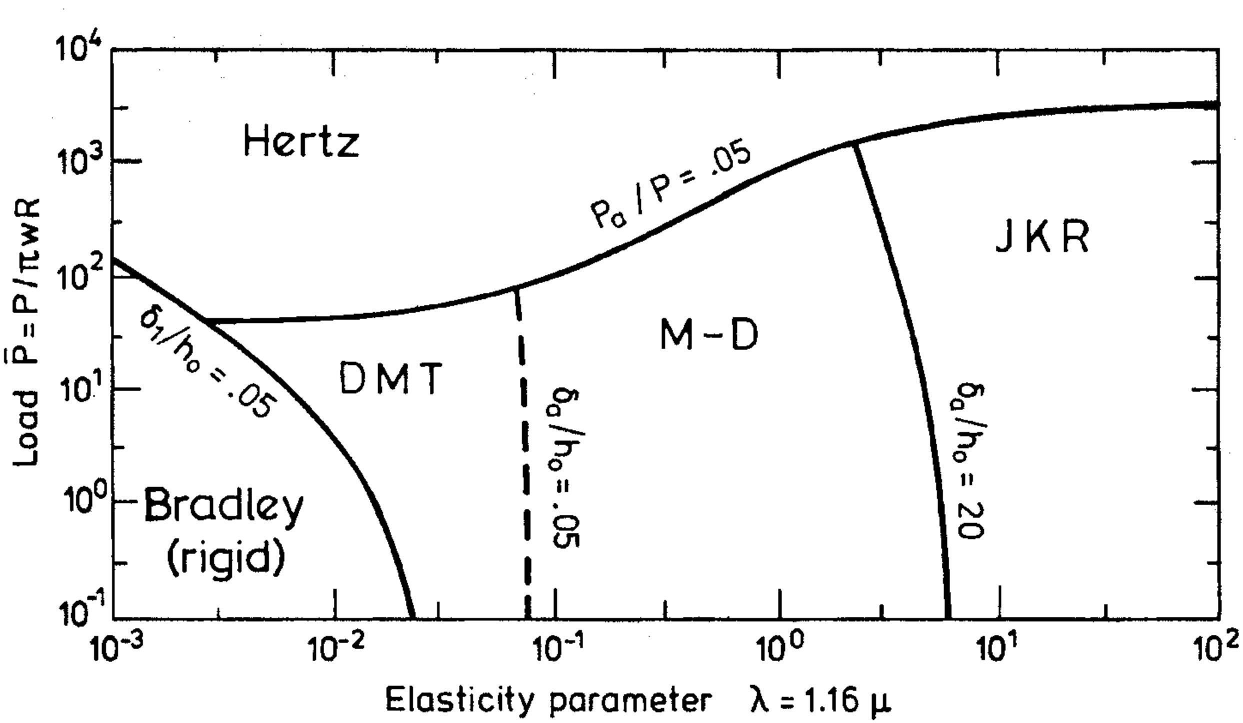
  ⇨ DMT Approximation

- $\mu > 10$ Low Young's modulus, high adhesion energy, large radii of curvature

  ⇨ JKR Approximation

Adhesion maps

Johnson, K. L. & Greenwood, J. A.
An Adhesion Map for the Contact of Elastic Spheres
Journal of Colloid and Interface Science, **1997**, *192*, 326-333



- **Elastomers** $\mu > 100$: JKR domain
- **Adhesion between polymer fibers:** μ ~ 10 or less : boundary of the JKR domain
- **Surface force apparatus (SFA):** μ ~ 50 with high adhesion: JKR regime
- **Atomic force microscopy (AFM)** : $\mu < 100$ tip radius 100 nm on a rigid surface: transition zone

Conclusions

- DMT or JKR depending on the materials properties
- $\lambda = w/E$ characteristic length scale for adhesion : nanometer range

Equilibrium conditions, isotrope and semi-inifinite bodies

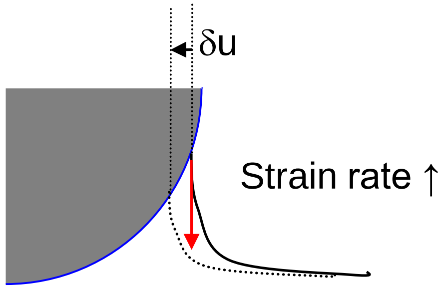
Other issues to be adressed

- Roughness
- Viscoelasticity
- Kinetics effects involved in adhesion (in relation to viscoelasticity)
- Thin coatings on substrates
- Friction

Kinetics effects in adhesion

Contact Mechanics ⟺ Fracture Mechanics

Crack propagation: dissipative effects in relation to characteristic viscoelastic times



- Adhesive contact of bulk viscoelastic bodies → intricate problem (Barthel *et al*, 2002, Hui *et al* 2002)
- Simplification in some specific contact situations where:

  - viscoelastic dissipation restricted to a narrow, highly strained region, at the edge of the contact

  and

  - the bulk response remains purely elastic

strain energy release rate can still be calculated using elastic contact theories.

$$G = w \longrightarrow w = G\,(da/dt)$$

Adhesion: role of viscoelasticity

$$G - w = w\,\phi\,(a_T v)$$

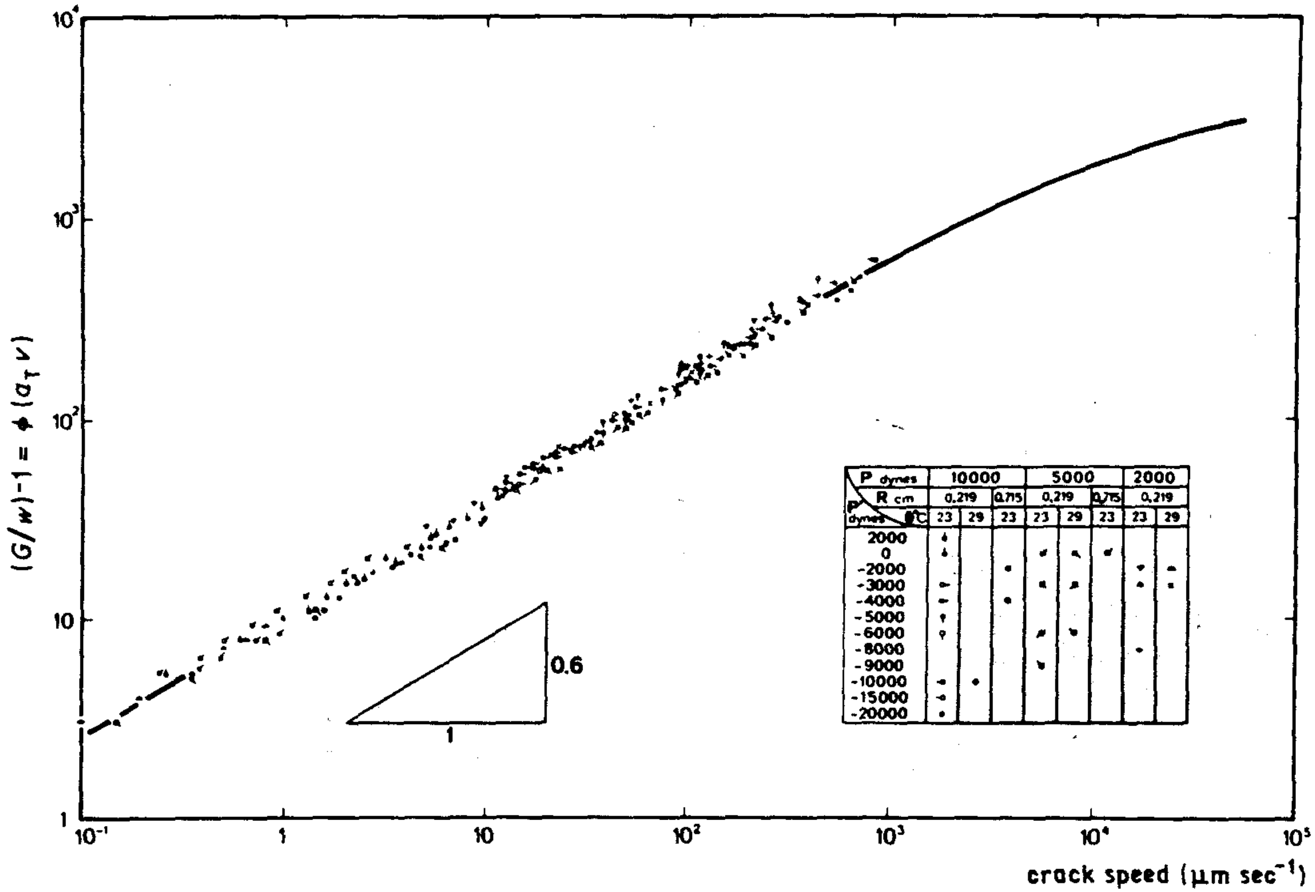
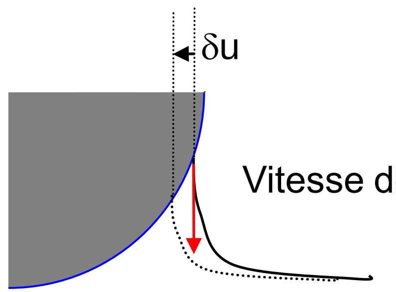


Figure 3 Reduced crack extension force against crack velocity for glass–polyurethane systems.

Fracture mechanics and the adherence of viscoelastic bodies.
Journal of Physics D: Applied Physics, **1978**, *11*, 1989-2023

Effets cinétiques en adhésion

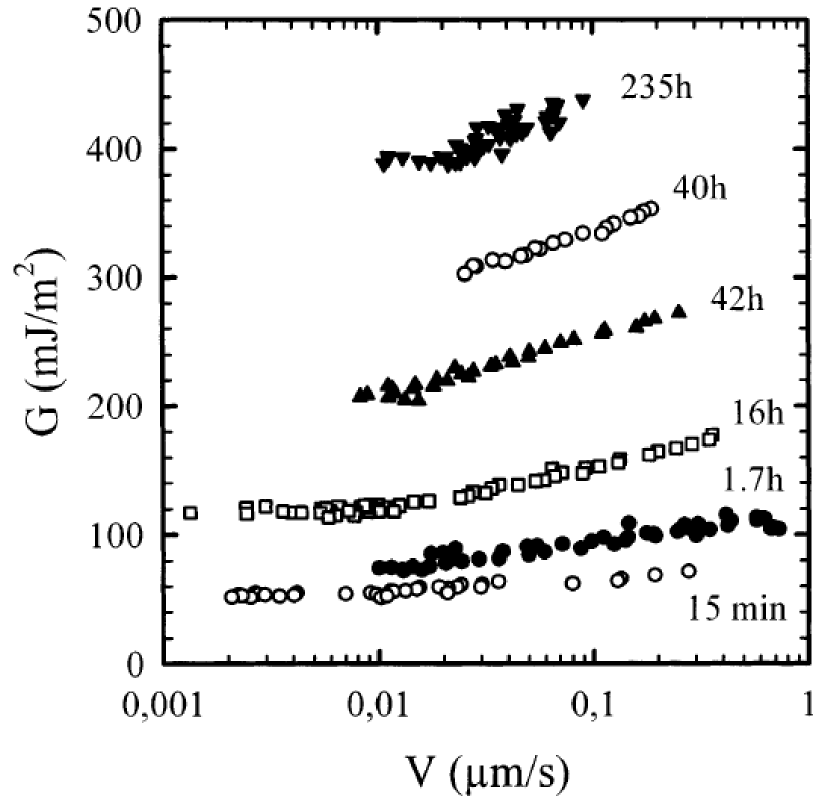
Dissipation viscoélastique en bord de contact

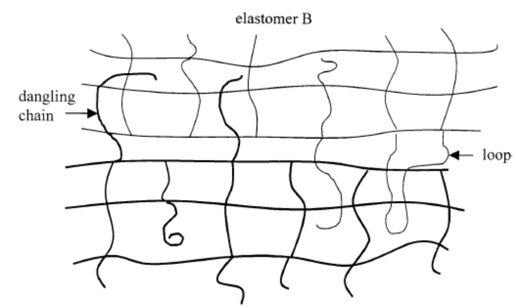


Vitesse de déformation↑

$G = w$ ⇒ $G = w(1+\phi(v))$

Adhésion PDMS/PDMS





- Adhésion de sphères de caoutchouc sur des substrats de PMMA rugue

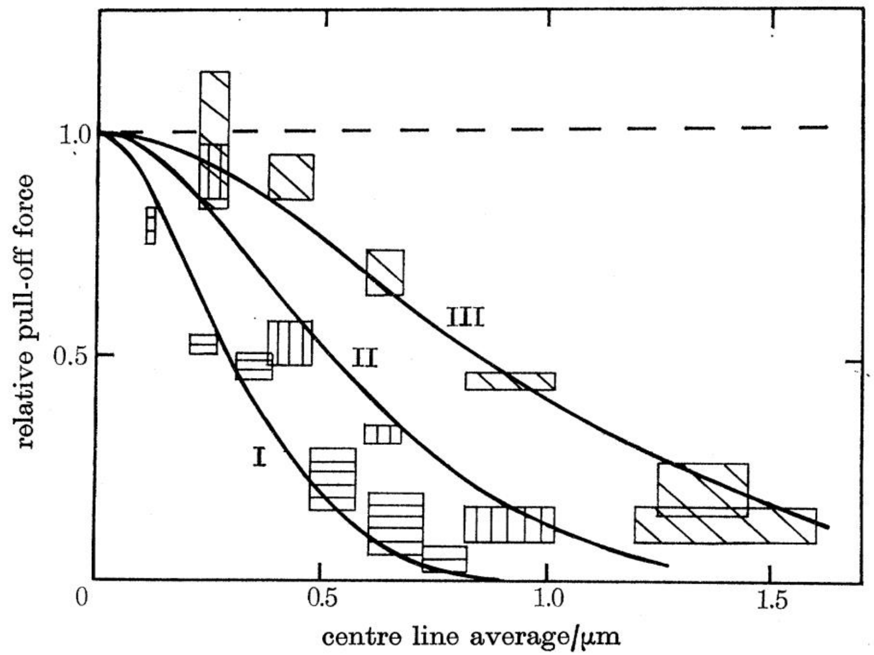
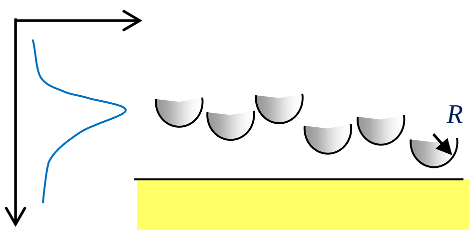


FIGURE 3. Relative pull-off force for smooth rubber spheres in contact with a flat Perspex surface as a function of the roughness (c.l.a.) of the Perspex. Effects of modulus, E, of the rubber: curve I, 2.4×10^6 N m^{-2}; curve II, 6.8×10^5 N m^{-2}; curve III, 2.2×10^5 N m^{-2}.



Paramètre d'adhésion: $\theta = \dfrac{E\sigma^{3/2}R^{3/2}}{R\gamma}$

Fibrilar bio-mimetic adhesives /I

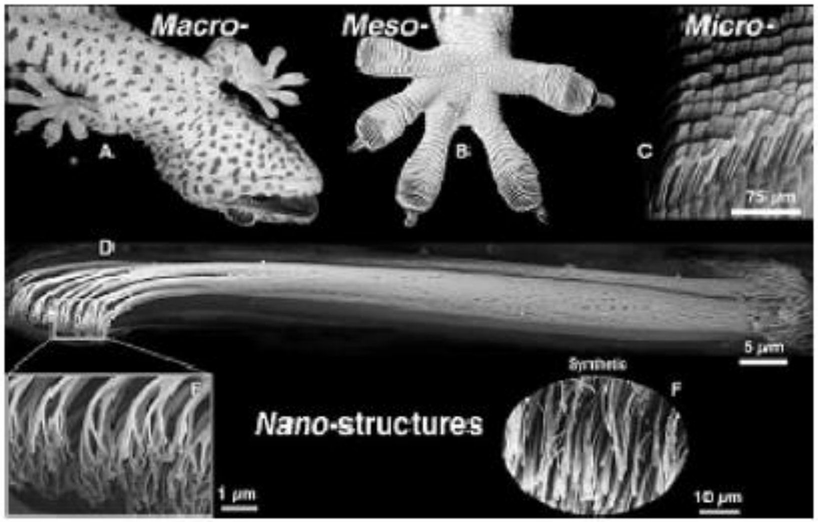


Fig. 1. Hierarchy of structure in the gecko toe attachment system (taken from Autumn[2]).
Reproduced from Autumn [2], with permission from the Materials Research Society.

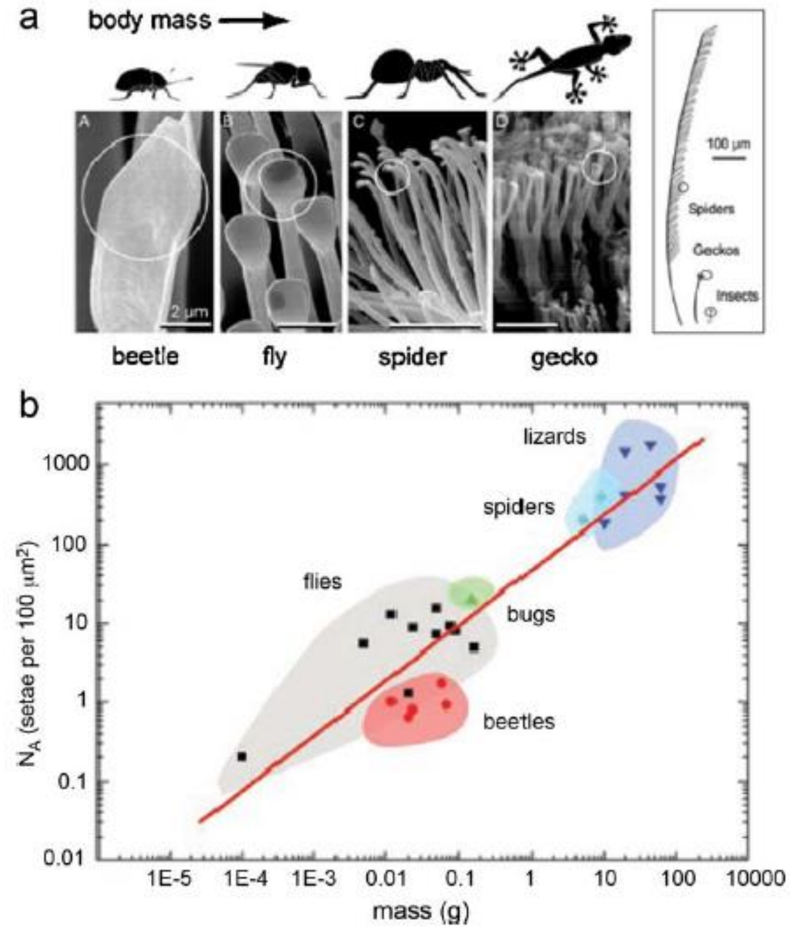


Fig. 3. (a) As body mass increases the fibril size decreases and the fibril density (number per unit area) increases. This is seen empirically in (b) over a large range in body mass and cutting across mechanisms that are 'dry' and 'wet'.
Reproduced with permission from [34] Copyright (2007) National Academy of Sciences, U.S.A.

Fibrilar bio-mimetic adhesives / II

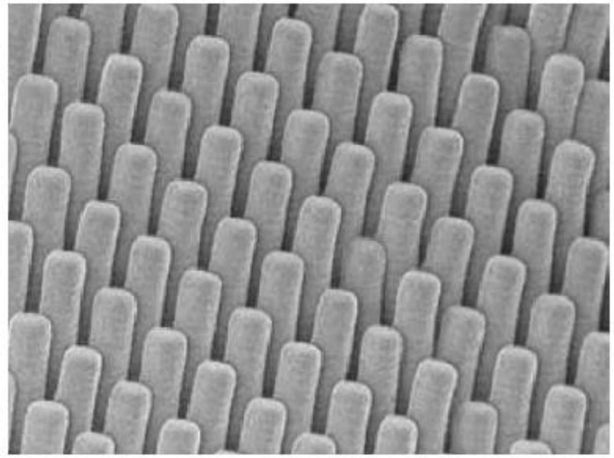


Fig. 4. An example of a simple 1-level fibrillar structure [29] fabricated by molding PDMS into microfabricated molds etched in a silicon wafer. Fibril diameter is 1 micron.

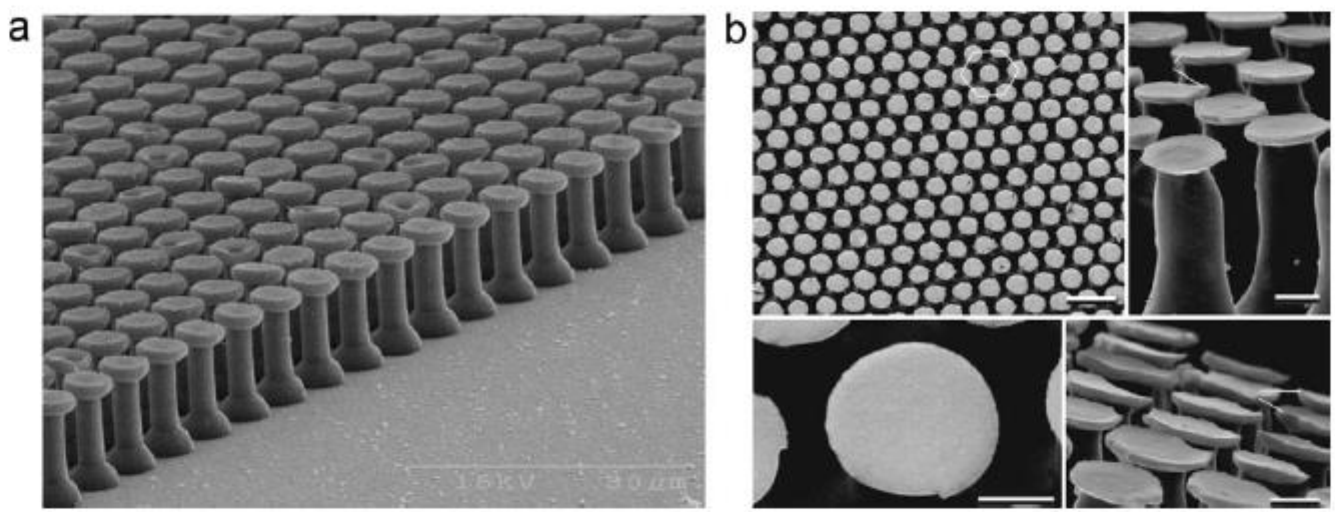


Fig. 5. Fibrillar arrays with a terminal design that removes edge singularities, strongly increasing the force it takes to remove a fibril from the surface. Figures from (a) reproduced with permission from [71] and (b) is reproduced from [72].

References

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Cambridge University Press, 1985

Contact, Adhesion and rupture of elastic solids
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Solid-State Sciences, Springer, 1999