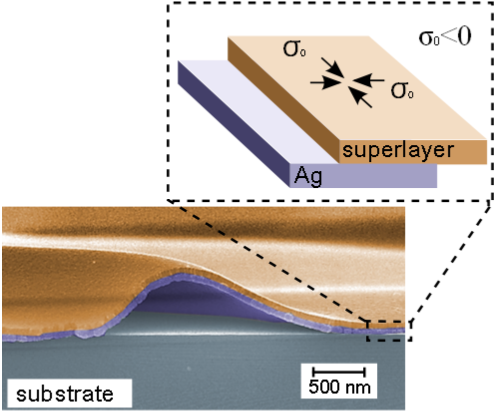
Thin films, buckles and instabilies

J.-Y. Faou, S. Grachev, G. Parry and E. Barthel

Surface du Verre et Interfaces / SIMAP

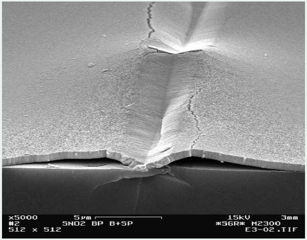
2013 / Material deformation – Les Houches



Buckles

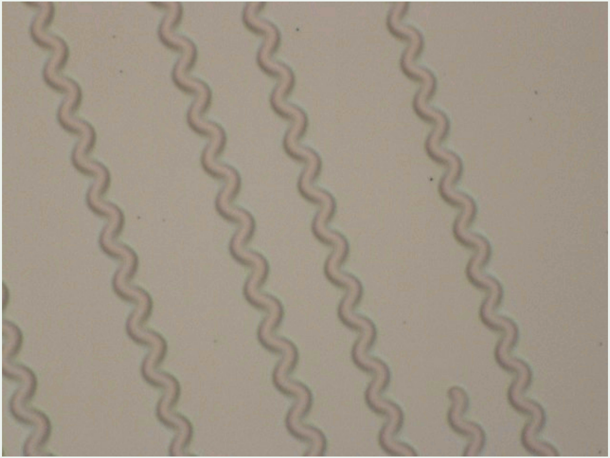
Thin films: multiple applications...

...and numerous issues !



CNRS
SAINT-GOBAIN

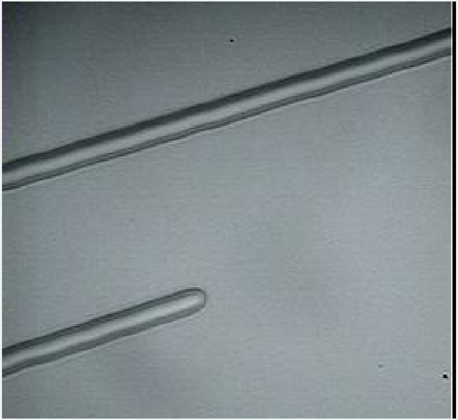
A telephone cord



Mo on Ag

- Why do most buckles propagate as telephone cords ?

Straight buckles



ZnO/Ag multilayer

- Why is a straight buckle stable on the sides ?
- Why does it propagate at its rounded tip ?

Content

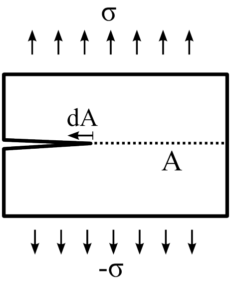
Energy release rate – Definition

- Definition

$$\mathcal{G} \equiv \left. \frac{d\mathcal{E}_{el}}{dA} \right|_{\delta}$$

- At equilibrium

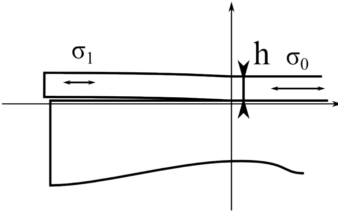
$$\mathcal{G} = \mathcal{G}_c$$



A crack under remote loading σ

Thin film / plate – Stretching

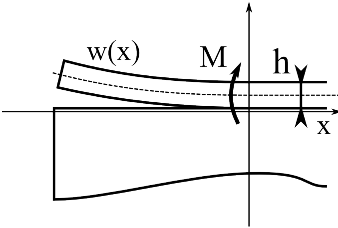
$$\mathcal{G} = \frac{h(\Delta\sigma)^2}{2E^\star} = \frac{1}{2}E^\star\epsilon^2 h$$



- stress variation $\Delta\sigma = \sigma_1 - \sigma_0 = E^\star\epsilon$
- plane strain modulus $E^\star = \frac{E}{(1-\nu^2)}$

Thin film / plate – Bending

$$\mathcal{G} = \frac{1}{2}\frac{M^2}{D} = \frac{1}{2}Dw''(0)^2$$

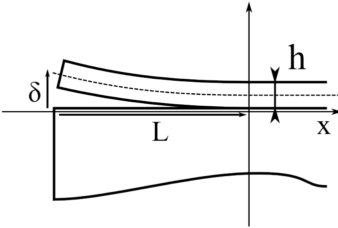


Cantilever beam

- bending moment $M = Dw''(0)$
- bending modulus $D = \frac{Eh^3}{12(1-\nu^2)}$

Thin film / plate – Bending

$$\mathcal{G} = \frac{1}{2}\frac{M^2}{D} = \frac{1}{2}Dw''(0)^2$$



Cleavage

Energy release rate for a DCB test

$$\mathcal{G} = \frac{3Eh^3}{8(1-\nu^2)}\left(\frac{\delta}{L^2}\right)^2$$

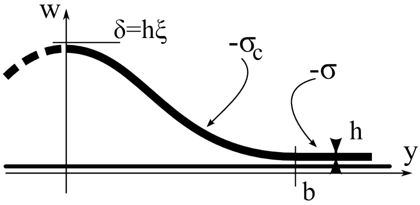
[Obreimov, 1930]

Content

Plate equilibrium
1D von Karman equation

$$D\,\frac{\partial^4 w}{\partial y^4} - N_{yy}\frac{\partial^2 w}{\partial y^2} = 0$$

$$\frac{dN_{yy}}{dy} = 0$$



- bending modulus

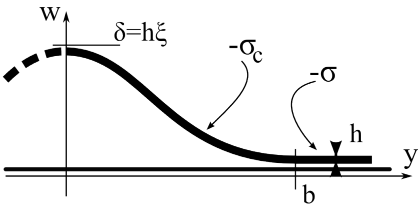
$$D = \frac{Eh^3}{12(1-\nu^2)}$$

- transverse loading

$$N_{yy} = \sigma_{yy} h$$

Plate equilibrium
1D von Karman equation

$$D\frac{\partial^4 w}{\partial y^4} - N_{yy}\frac{\partial^2 w}{\partial y^2} = 0$$
$$\frac{dN_{yy}}{dy} = 0$$



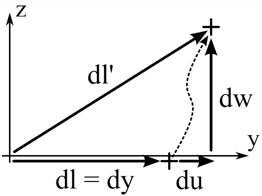
with $w = \frac{1}{2}\delta\left[1+\cos\left(\frac{\pi y}{b}\right)\right]$ we find $\frac{D\pi^2}{b^2} - \sigma_c h = 0$

Buckling stress

$$\sigma_c = \frac{\pi^2 D}{b^2 h}$$

Strains in the plate – non linear

$$\epsilon_y = \frac{du}{dy} + \frac{1}{2}(\frac{dw}{dy})^2$$

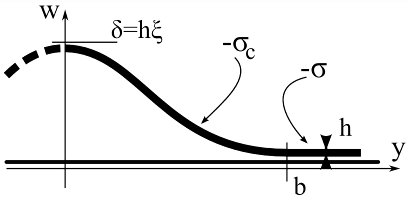


elongation $dl'^2 = (dy + du)^2 + dw^2$ so that

$\frac{dl'}{dl} \simeq 1 + \frac{du}{dy} + \frac{1}{2}\left(\frac{dw}{dy}\right)^2$

Strains in the plate – non linear

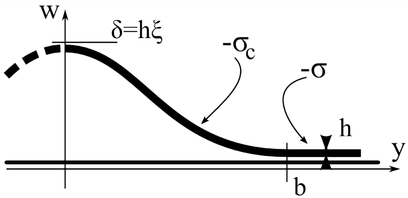
$$\epsilon_y = \frac{du}{dy} + \frac{1}{2}(\frac{dw}{dy})^2$$



$2b\frac{\Delta\sigma}{E^\star} = \frac{b}{2}\frac{\pi^2\delta^2}{4b^2}$ with plane strain modulus $E^\star = \frac{E}{(1-\nu^2)}$

Strains in the plate – non linear

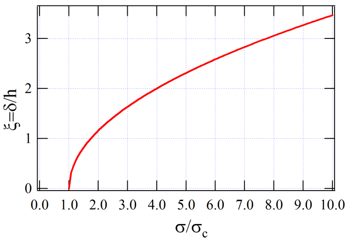
$$\epsilon_y = \frac{du}{dy} + \frac{1}{2}(\frac{dw}{dy})^2$$



$2b\frac{\Delta\sigma}{E^\star} = \frac{b}{2}\frac{\pi^2\delta^2}{4b^2}$ with plane strain modulus $E^\star = \frac{E}{(1-\nu^2)}$

Buckle amplitude

$$\xi^2 = \frac{4}{3}\left(\frac{\sigma}{\sigma_c} - 1\right)$$



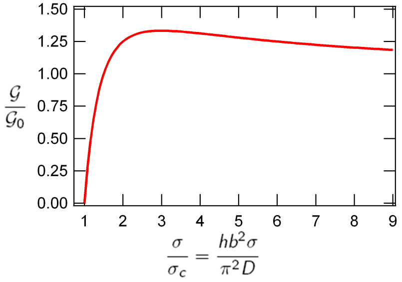
Euler buckle

- Energy release rate

$$\mathcal{G} = \frac{1}{2}\frac{(\sigma - \sigma_c)(\sigma + 3\sigma_c)}{E^\star}$$

- Total elastic energy stored

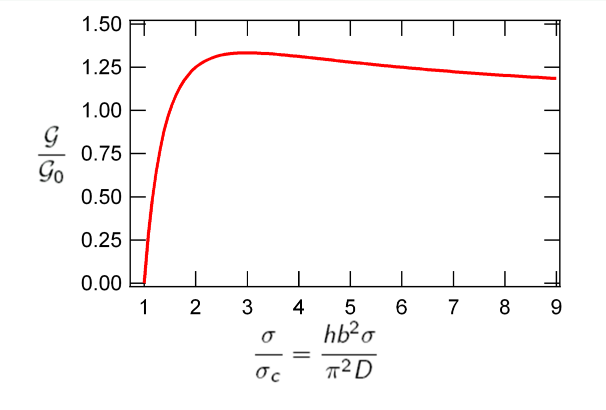
$$\mathcal{G}_0 = \frac{1}{2}\frac{h\sigma^2}{E^\star}$$



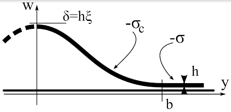
- ERR $\mathcal{G} = \frac{1}{2}\frac{M^2}{D} + \frac{1}{2}\frac{h\Delta\sigma^2}{E^\star}$
- moment $M = \frac{\pi^2}{2} D \frac{\delta}{b^2}$
- traction $\Delta N = h\Delta\sigma = \frac{\pi^2}{16} h \frac{E^\star \delta^2}{b^2}$

[Hutchinson and Suo, 1992]

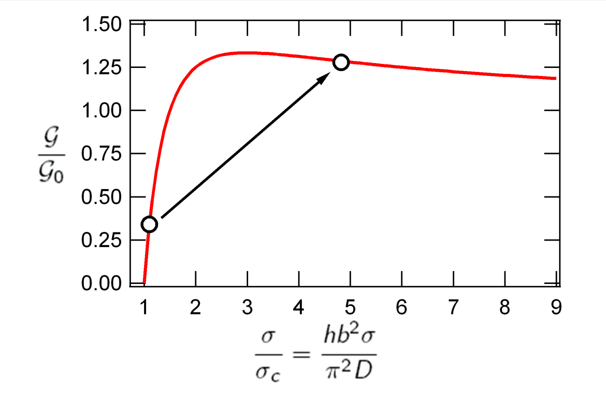
Euler buckle – stability

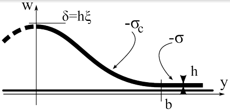


[Hutchinson and Suo, 1992]



Euler buckle – stability

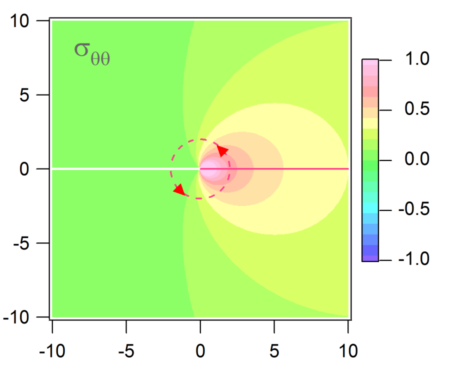




[Hutchinson and Suo, 1992]

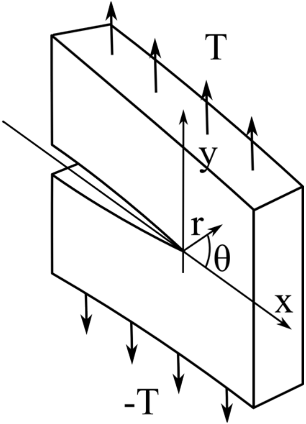
Content

Crack tip stress field – mode I



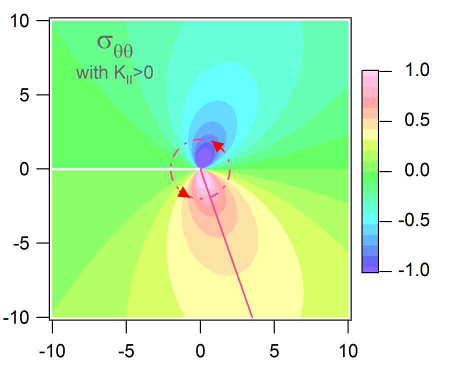
Orthoradial stress distribution $\tilde{\sigma}(\theta)$

$$\sigma(r,\theta) = \frac{K_I}{\sqrt{2\pi r}}\tilde{\sigma}(\theta)$$

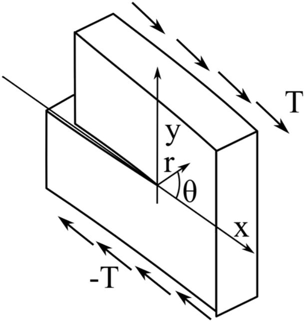


[Erdogan and Sih, 1963]

Crack tip stress field – mode II



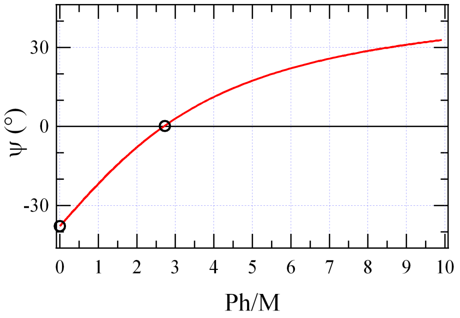
Orthoradial stress distribution $\tilde{\sigma}(\theta)$

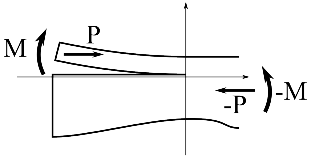


$$\sigma(r,\theta) = \frac{K_{II}}{\sqrt{2\pi r}}\tilde{\sigma}(\theta)$$

[Erdogan and Sih, 1963]

Edge crack for a thin film

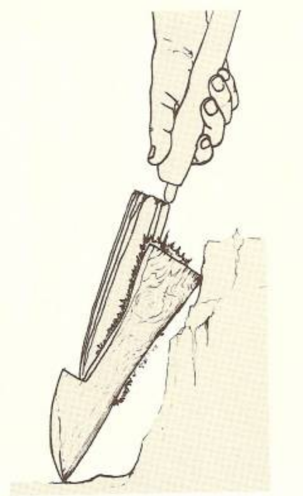




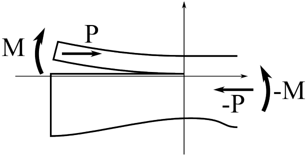
Mode mixity for an edge crack

Mode mixity angle $\tan(\psi) = \frac{K_{II}}{K_I}$ [Hutchinson and Suo, 1992]

Edge crack for a thin film



Pressure blade making [Pelegrin, 2005]

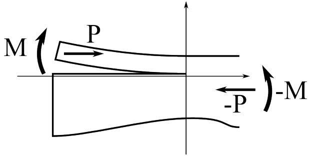


Mode mixity for an edge crack

Edge crack for a thin film

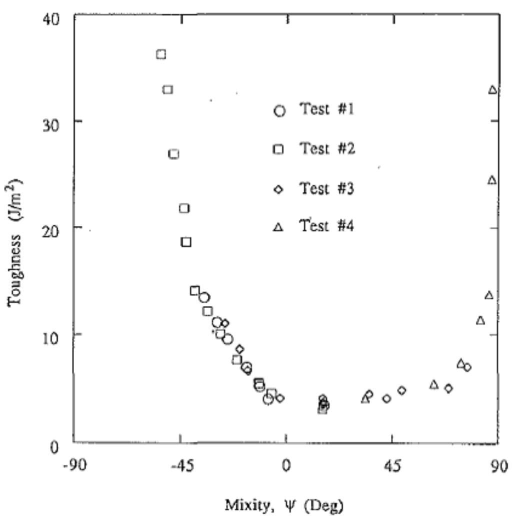
$$K_I = \frac{\cos\omega}{\sqrt{2}} P h^{-\frac{1}{2}} + \sqrt{6}\sin\omega M h^{-\frac{3}{2}}$$
$$K_{II} = \frac{\sin\omega}{\sqrt{2}} P h^{-\frac{1}{2}} - \sqrt{6}\cos\omega M h^{-\frac{3}{2}}$$

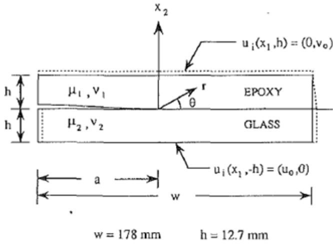
with $\omega \simeq 52^\circ$



Mode mixity for an edge crack

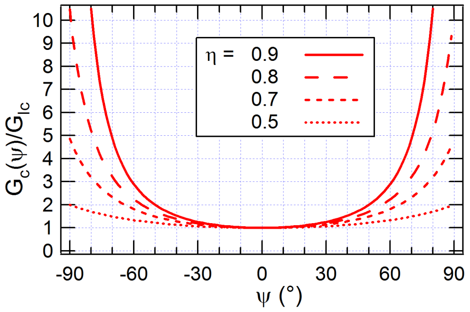
Mixed Mode Toughness

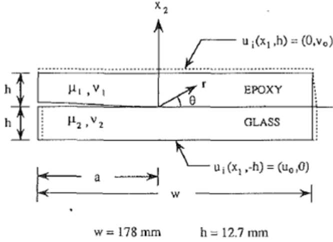




[Chai and Liechti, 1992]

Mixed Mode Toughness

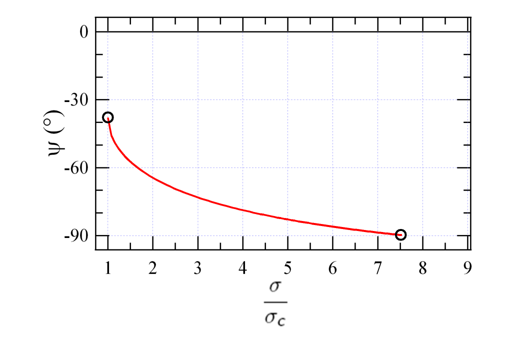




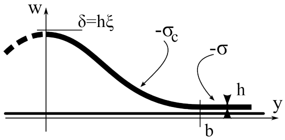
[Chai and Liechti, 1992]

$$f(\psi) = \frac{\mathcal{G}_c(\psi)}{\mathcal{G}_{Ic}} = 1 + \tan(\eta\psi)^2$$

Impact for Euler buckle



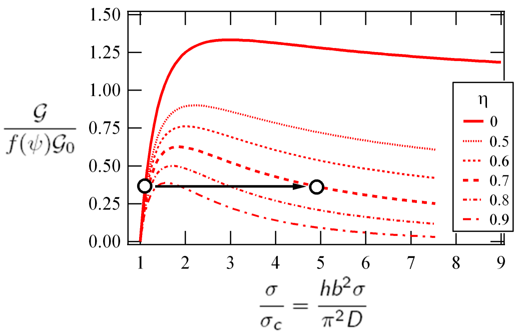
Mode mixity



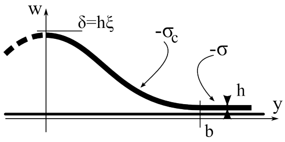
$$\tan\psi = \frac{\cos\omega + \frac{\sqrt{3}}{4}\xi\sin\omega}{-\sin\omega + \frac{\sqrt{3}}{4}\xi\cos\omega}$$

with $\omega \simeq 52°$

Impact for Euler buckle



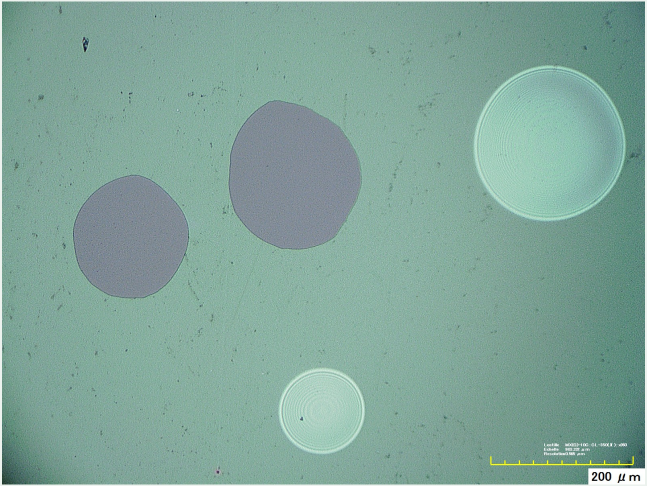
Mode corrected ERR



- Straight buckles CAN be stable on the sides

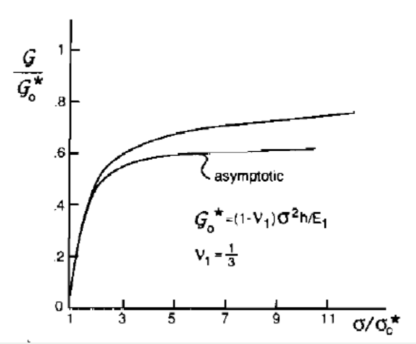
Content

Buckles, spalls...

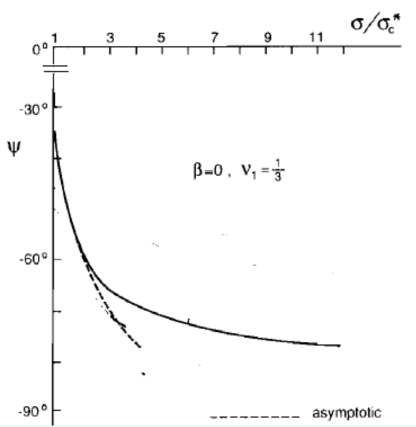


A. Benedetto (SGR)

Energy release rate



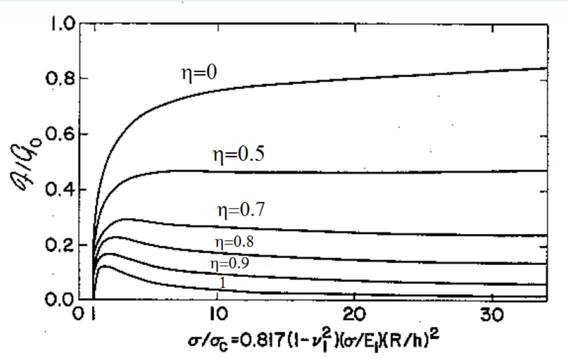
Mode mixity



Circular buckle

Hutchinson et al. [1992]

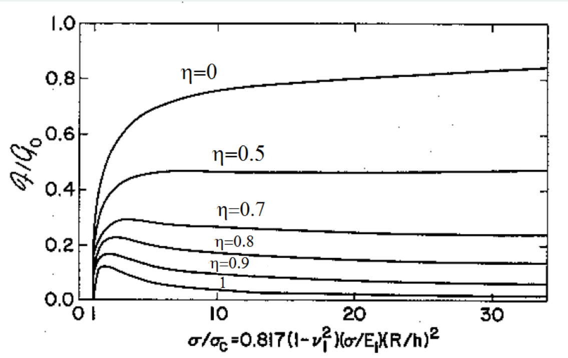
Circular buckle – mode adjusted ERR

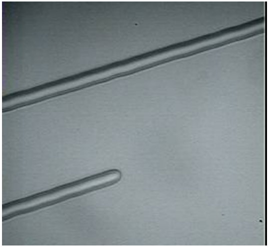


- circular buckles are seldom stable
- a straight buckle propagates at its rounded tip

Hutchinson et al. [1992]

Circular buckle – mode adjusted ERR

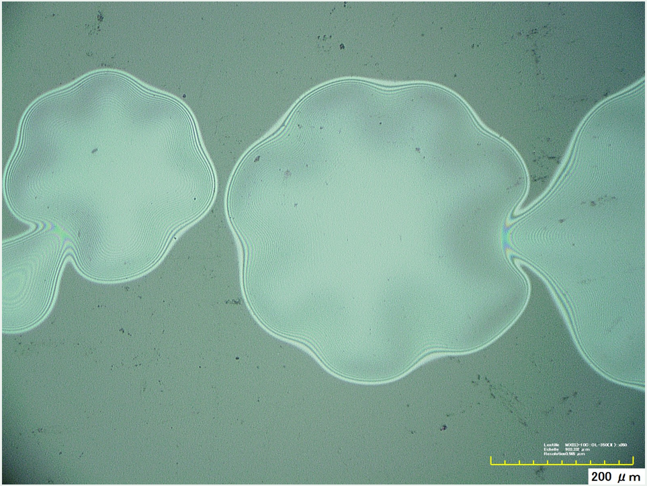




- circular buckles are seldom stable
- a straight buckle propagates at its rounded tip

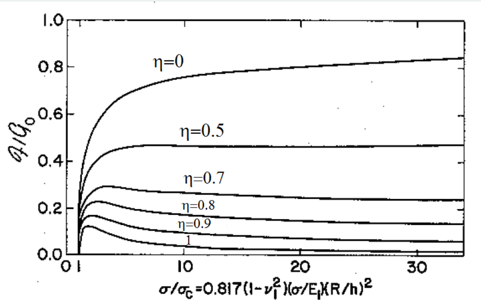
Hutchinson et al. [1992]

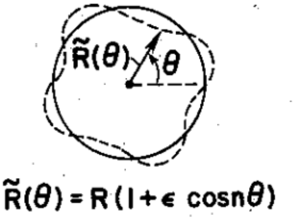
... and scalloped buckles!



A. Benedetto (SGR)

Circular buckle – Instability

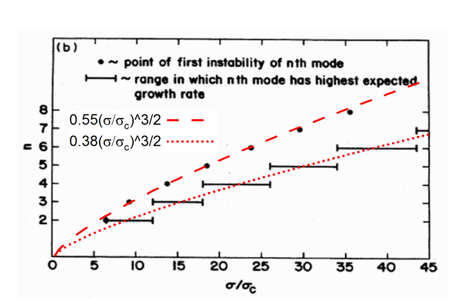


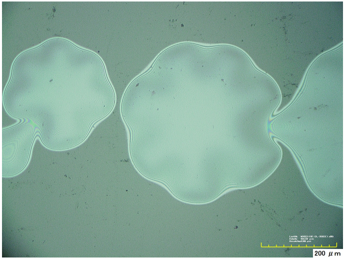


- Perturbation: $u(r) = u^{(0)}(r) + \epsilon u^{(1)}(r)\cos(n\theta)$
- Response: $F = F^{(0)}(\sigma/\sigma_c) + \epsilon F^{(1)}(\sigma/\sigma_c, n)\cos(n\theta)$

Hutchinson et al. [1992]

Circular buckle – Instability





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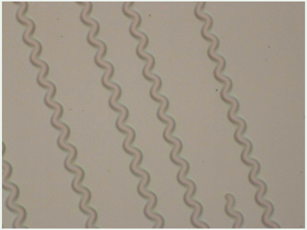
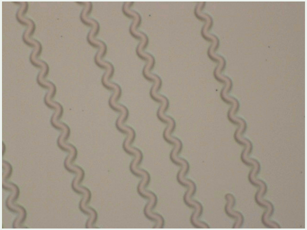
Hutchinson et al. [1992]

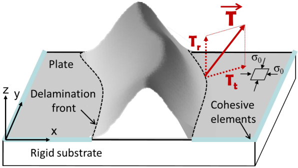
Content

FEM simulation –Cohesive zone model



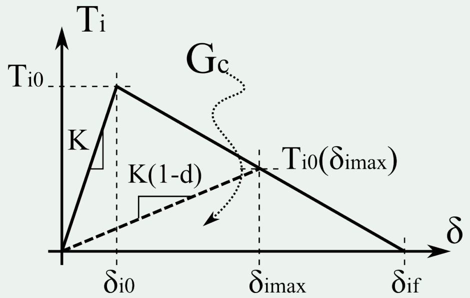
FEM simulation –Cohesive zone model

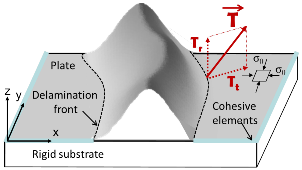




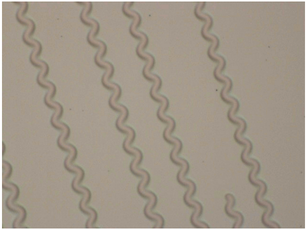
Cohesive elements

FEM simulation –Cohesive zone model

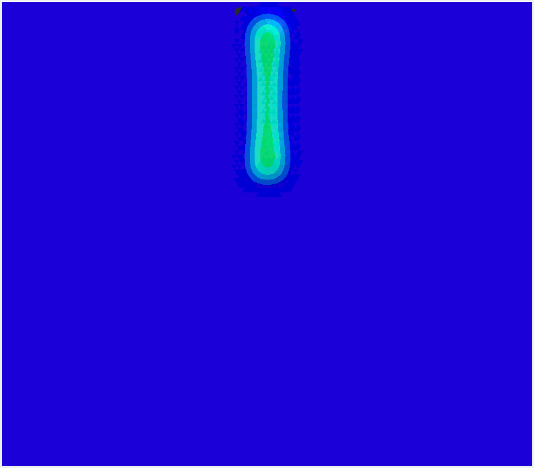




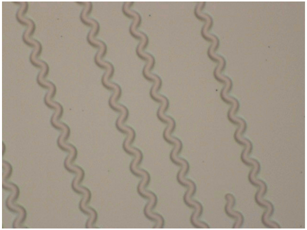
Cohesive elements



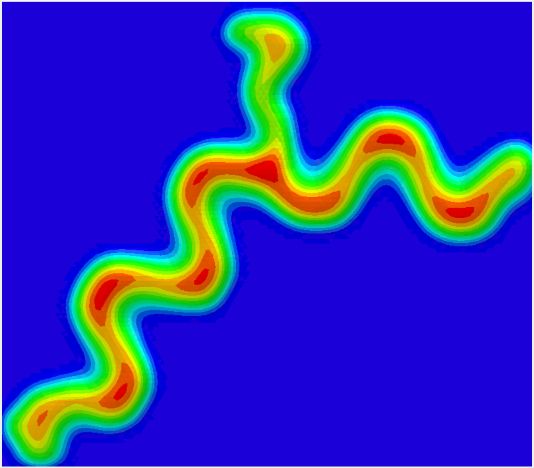
Telephone cord – simulation



[Faou et al., 2012]



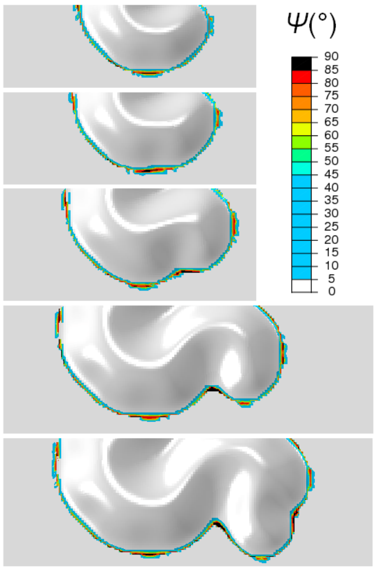
Telephone cord – simulation



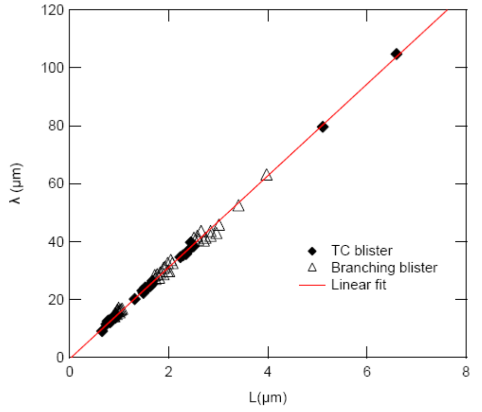
[Faou et al., 2012]

Telephone cord – mode mixity

[Faou et al., 2012]

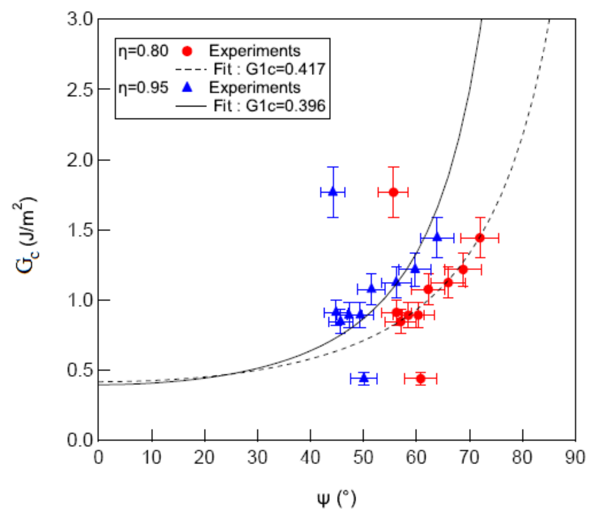


Telephone cord – wavelength

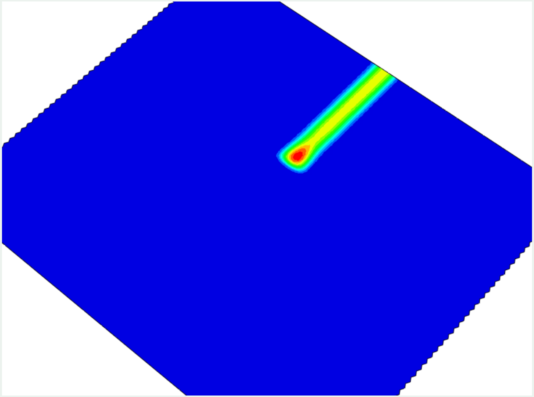


$$\frac{\sigma}{E^{\star}\left(\frac{h}{L}\right)^{2}} \equiv \frac{1}{\sqrt{\frac{G_0}{G_{Ic}}-1}}$$

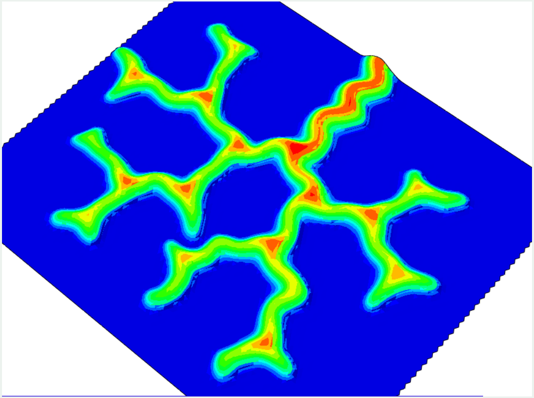
Experiments – mixed mode toughness from λ



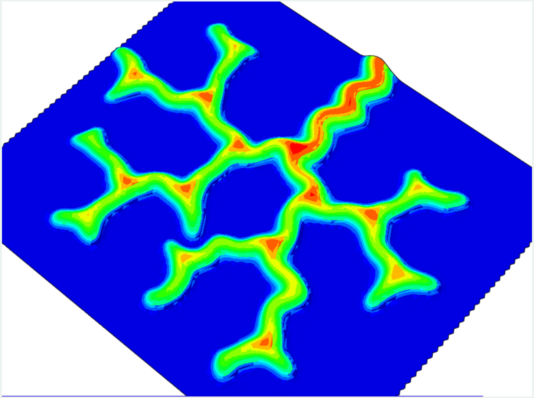
Branched telephone cord – simulations

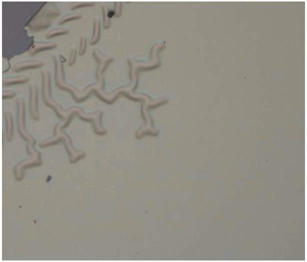


Branched telephone cord – simulations

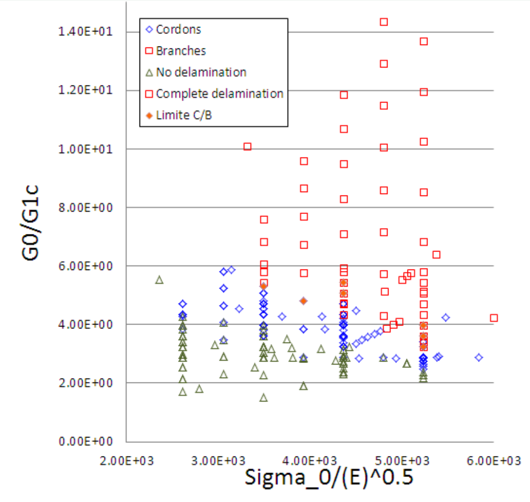


Branched telephone cord – simulations





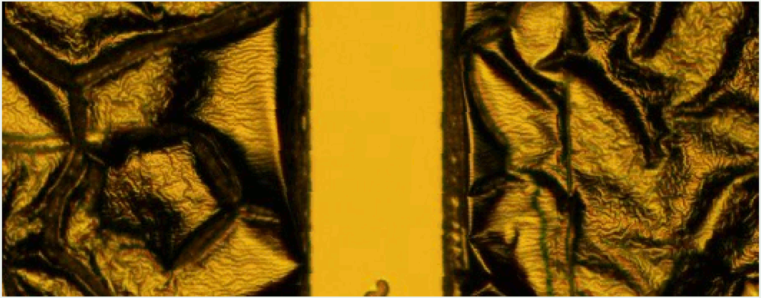
A phase diagram



Conclusion

For thin films with large compressive stresses

- mode-dependent interface toughness
- couples with non-linear plate buckling deformation
- drives configurational instability



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